Laplace Equation

$$q = -\nabla\phi$$

$$\nabla\cdot r = -\nabla(\nabla\phi) = \nabla^2\phi = 0$$

Spherical Polar cordinates

$$x = r\cos\phi\sin\theta$$

$$y = r\sin\phi\sin\theta$$

$$z = r\cos\theta$$

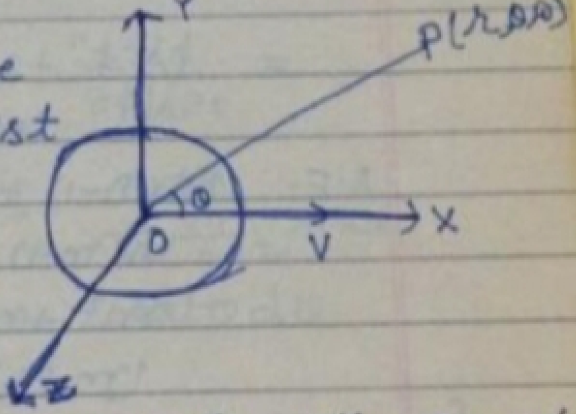
$$x^2+y^2+z^2 = r^2$$

$$\phi = \tan^{-1}\left(\frac{y}{x}\right)$$

$$\theta = \tan^{-1}\left(\frac{x^2+y^2}{z^2}\right)$$

§ 8.2 (P-400) Motion of a sphere in an infinite mass of liquid at rest at infinity :—

since the motion is irrotational so the velocity Potential exist then



$$r = -\nabla\phi \Rightarrow \nabla\cdot q$$

$$= -\nabla(\nabla\phi) = 0$$

$$\Rightarrow \nabla^2\phi = 0$$

In sphrical Polar cordinate, the Equation can be experred as

$$\frac{\partial}{\partial r}\left(r^2\frac{\partial\phi}{\partial r}\right) + \frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\phi}{\partial\theta}\right) = 0$$

The boundary condition are given as

(i) $\left(-\frac{\partial\phi}{\partial r}\right)_{r\to\infty} = 0$

(2) $\left(\frac{-\partial\phi}{\partial r}\right)_{r=a} = V\cos\theta$

the surface condition can be satisfied for all θ and ϕ is proportional to the axis symmetric surface harmonic.

we assume the

$$\text{Let } \phi = f(r)\cos\theta$$

$$\frac{\partial}{\partial r}\left[r^2 f' \cos\theta\right] + \frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left[-\sin\theta \cdot f(r)\sin\theta\right] = 0$$

$$= \frac{\partial}{\partial r}\left(r^2 f' \cos\theta\right) - \frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(f\sin^2\theta\right) = 0$$

$$= \frac{\partial}{\partial r}\left(r^2 f'\right)\cos\theta - \frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(f\sin^2\theta\right) = 0$$

$$= \left(r^2 f'' + 2rf'\right)\cos\theta - \frac{1}{\sin\theta}\left[2f\sin\theta\cos\theta\right] = 0$$

$$= \left(r^2 f'' + 2rf'\right)\cos\theta = 0$$

$$\cos\theta \neq 0 \quad \text{as } \theta \neq \frac{\pi}{2}$$

A.E. $(D(D-1) + 2D - 2)f = 0$

$$m(m-1) + 2m - 2 = 0$$

$$m^2 + m - 2 = 0$$

$$(m+2)(m-1) = 0$$

$$m = 1, -2$$

$$f(r) = Ar + \frac{B}{r^2}$$

$$\phi = \left(Ar + \frac{B}{r^2}\right)\cos\theta$$

$$\frac{\partial\phi}{\partial r} = \left(A - \frac{2B}{r^3}\right)\cos\theta$$

since the space derivative ϕ must vanishes at ∞, so

$A\cos\theta = 0$

$\Rightarrow A = 0, \cos\theta \neq 0$

By second condition

$$-\left(A - \frac{2B}{a^3}\right)\cancel{\cos\theta} = V\cancel{\cos\theta}$$

$$-(Aa^3 - 2B) = Va^3$$

$$2B = Va^3 \qquad \text{since } A = 0$$

$$B = \frac{1}{2}Va^3$$

$$\phi = \frac{Va^3}{2r^2}\cos\theta$$

And $\psi = -\frac{1}{2}\frac{Va^3}{r}\sin\theta$

Lines of flow:-

$$\frac{dr}{\partial\phi/\partial r} = \frac{r\,d\theta}{\partial\phi/r\partial\theta}$$

$$\frac{dr}{-\frac{Va^3}{r^3}\cos\theta} = \frac{r\,d\theta}{-\frac{1}{r}\frac{Va^3}{2r^2}\sin\theta}$$

$$\frac{1}{r}\,dr = 2\cot\theta\,d\theta$$

Integrating $\log r = \log c + \log \sin^2\theta$

$$\boxed{r = c\sin^2\theta}$$

this is Represent the Equation of the lines of flow.

— x —

Complete unit I